

Vector Identity (3)

$$\nabla(fg) = f(\nabla g) + g(\nabla f)$$

Proof

$$\begin{aligned}\nabla(fg) &= \sum_{i=1}^3 \delta_i \frac{\partial}{\partial x_i} (fg) \\ &= \sum_{i=1}^3 \delta_i \left(\frac{\partial f}{\partial x_i} g + f \frac{\partial g}{\partial x_i} \right) \\ &= \sum_{i=1}^3 \delta_i \frac{\partial f}{\partial x_i} g + \sum_{i=1}^3 \delta_i f \frac{\partial g}{\partial x_i} \\ &= g \left(\sum_{i=1}^3 \delta_i \frac{\partial f}{\partial x_i} \right) + f \left(\sum_{i=1}^3 \delta_i \frac{\partial g}{\partial x_i} \right) \\ &= g(\nabla f) + f(\nabla g) \\ &= f(\nabla g) + g(\nabla f)\end{aligned}$$